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The development of mathematical competences in Hungarian teacher training education*

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Abstract

In this paper we present research results on the assessment of K1-4 (pupils from age 7 to age 10) teacher training student's mathematical knowledge and competences as one of the most important parameters of school teaching quality. Teachers' abilities in grades K1-4 are among the most important in-school factors influencing the quality of pupils' learning. A large-scale longitudinal study was conducted in which the elementary mathematical knowledge and skills of a group of teacher training students from 5 different institutes was assessed by means of a paper and pencil test that was administered both at the beginning and at the end of their second year mathematical course in the 2016/2017 academic year. This course is a methodical course in some institutions, which has an essential influence on the final results of this research. The 27-item-test covered the new standards for mathematics in the K1-4 elementary school curriculum. We have observed that those students coming from institutions providing separate methodical courses can gain better knowledge in explaining simple mathematical relations and notions than

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those students whose institutions do not provide methodical background in specific courses.

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1. Introduction and problem statement

Over the past three decades, higher education in OECD countries has changed profoundly. Not only has participation soared but student populations have become much more diverse. In response, systems have expanded and new providers with new offerings have emerged. This long period of expansion has distracted attention from the actual outcomes of higher education, but OECD countries are now looking more closely at how to ensure quality in education. As pointed out by the OECD teachers' review in 2005 (see [4]), education systems need to invest in intensive teacher education and training if teachers are expected to deliver high-quality outcomes. This also refers to the ECEC sector [5]: specific knowledge, skills and competencies are expected of ECEC practitioners. What does it mean to be a competent mathematics teacher? The answer is not simple: competent mathematics teacher has been trained in mathematics and receives some additional pedagogical and didactical training. Furthermore, teachers have to learn mathematics in ways that are specifically focused on teaching at a certain level, which is called "pedagogical content knowledge" by Shulman [3]. To possess mathematical competences means having knowledge of understanding, doing, using, and having a well-founded opinion about mathematics in variety of situation and contexts where mathematics plays or can play a role. Niss identified eight main constituents in that competence in [1], each of which is called a mathematical competence:

- The ability to ask and answer questions in and with mathematics:
  - Mathematical thinking competence: mastering mathematical modes of thought
  - Problem handling competence: formulating and solving mathematical problems
  - Modelling competence: being able to analyse and build mathematical models
  - Reasoning competence: being able to reason mathematically
- The ability to deal with mathematical language and tools:
  - Representation competence: being able to handle different representations of mathematical entities
  - Symbol and formalism competence: being able to handle symbol language and mathematical formalism

– Communication competence: being able to communicate in, with and about mathematics
– Aids and tools competence: being able to make, use of and relate to aids and tools of mathematics.

A competent mathematics teacher is someone who is able to help his or her students in an effective and efficient way to build and develop their mathematical competencies. A competent mathematics teacher must be mathematically and methodically competent as well in the outlined sense. This study tries to explore to what extent the teacher training students possess the mathematical competences mentioned above.

The following three domains received a primary focus in our research programme: knowledge of subject matter, pedagogical content knowledge, and knowledge of learners' cognition, based on the partition published by Shulman in [3] (see also: [2]). Applied to the domain of mathematics education, these three domains of knowledge can be described as follows.

First, subject-matter knowledge includes mastery of the key facts, concepts, principles and explanatory frameworks, procedures and problem solving techniques and strategies within the given domain of instruction. Crucial in this respect is also the level of teachers' understanding of the domain. The second category of teachers' knowledge can be defined as "knowledge of subject matter for teaching" ([3, p. 9]). It consists of an understanding of how to represent specific subject matter appropriately to the diverse abilities and interests of learners. It includes several issues, such as knowledge of mathematics lesson scripts and mathematics teaching routines, knowledge about various problem types, graphical representations, etc. that are best suited to introduce particular mathematical notions and skills to pupils. Furthermore, knowledge of instructional materials (textbooks, manipulatives, software, tests, etc.) available for teaching various mathematical topics is also essential. Third, there is teachers' knowledge of how students think and learn with respect to mathematics. This third component consists of the teachers' knowledge of the mathematical concepts and procedures that students bring to the learning of a topic, the misconceptions and buggy procedures that they may have developed, and the stages of understanding and skill that they are likely to pass through in the course of gaining mastery of it. In the Hungarian universities of teacher training the first years of training is primarily theoretical. The proportion of hours spent on theory decreases during the 4 years of the training, while gradually more time becomes available for practice. As far as mathematics education is concerned, the three major components of professional domain-specific knowledge discussed above (i.e., mathematical competence, pedagogical content knowledge, and knowledge of students) are typically addressed in one course. The undergraduate courses are followed by the Mathematics Methodology course, where students can learn how to teach each topic. However, there are institutions where there is no dedicated methodical course, but every course has a methodical aspect as well.

Overall, there are substantial differences between the institutes in terms of the relative proportion of instruction time that is devoted to each of these three components, the level of integration of these components, and, what exactly is being taught and learnt during this course. In Hungary there is no entrance exam or any other form of selection at the beginning of programs for higher education, including the training of elementary school teachers. Anyone who finished secondary school successfully and received his or her matriculation, can enter this teacher-training program. As a consequence, many students drop out during the first year of training or do not succeed in their exams. The low level of mathematical content knowledge and skills of students who want to become an elementary school teacher is increasingly being considered a major issue of concern among policy makers, curriculum developers, and teacher trainers involved in the training of future elementary school teachers. This growing concern was the major reason to set up this study.

2. The survey

A survey of mathematical competencies among K1-4 primary school teacher students of Hungarian teacher training institutions was held during the 2016/2017 academic year. Participants were 177 teacher training students who started mathematical methodological course (in those institutions where this course exists). These teacher students belonged to 5 institutions for teacher training. A paper-and-pencil mathematics test was administered to these 177 students during the first week of the academic year. At the end of the course a parallel version of this pretest was administered. The test was divided in six subsets differing in terms of the curricular subdomains and of the cognitive operations being addressed by the item.

2.1. Survey materials

The starting point for the construction of the mathematical competence pretest and posttest were the new standards for elementary education that have become operational in the Hungary since 2012. These standards cover all domains of the curriculum, including mathematics, and state the competencies that children should possess at the end of elementary school. The Hungarian curriculum standards for mathematics education are officially classified into different categories. Starting from this classification, we decided to divide the standards into six subdomains that were formed by combining a content and a cognitive dimension. The content dimension divided the mathematical content into two categories: Numbers and Arithmetics; Measurement and Geometry. Because more than half of the standards refers to the content domain of number and Arithmetics, we decided to combine the two other content domains (measurement and geometry) into one single domain. The cognitive dimension distinguished among three categories: Declarative knowledge, Procedural knowledge, and Strategic and problem solving skills. This resulted in a classification scheme consisting of six subdomains. It is worth noting, that none of the items required mathematical knowledge or skills beyond the content of the mathematics curriculum of the elementary school in Hungary. Nevertheless, the test contained several items that required good understanding of these elementary school mathematical notions and/or the application of problem solving strategies for using these mathematical notions in contextual problems. In a further stage of the project, we also constructed a parallel version of the first mathematics test. This parallel test—to be used as posttest—contained problems that were isomorphic to the problems from the pretest, but that were different in terms of superficial task characteristics (i.e., the concrete numbers used, the names of the persons and objects in the word problems, etc.).

2.2. Survey procedure

Shortly before the start of the academic year 2016/2017, copies of the pretest were sent to the 5 participating institutes, together with specific instructions on how the test had to be administered to the student teachers and how the completed forms had to be returned to the researchers. The pretest was administered in all institutes during the first week of the academic year 2016/2017. The administration of the test took 90 min. At the beginning of the pretest session the teacher trainer introduced the test and motivated the student teachers to do their best. At the same time, it was emphasized that the results would not be used for evaluative purposes within the context of their teacher training. At the end of the pretest session all copies were returned to the researchers who scored all test sheets according to strict criteria, leading to either 0-5 points for each of the 27 items. Most items were scored dichotomously on correctness of the answer. For the other items credit was given to partially correct answers. These could either refer to a correct response on a subset of questions or problems that were framed within one item, or to a partially correct response to a simple item (like when solving correctly the first step of a multi-step word problem). The organization and administration of the posttest was done in the same way at the same institutions as it happend in the case of the pretest.

2.3. Typical survey questions

Here we provide some of the examples that were included in the study tests.

- Numbers and Arithmetics – Declarative knowledge: What digit represents the tens, and what digit represents the thousands in the number 654,372?
- Numbers and Arithmetics – Procedural knowledge: Solve the operation! $3717 + 8635$
- Numbers and Arithmetics – Strategic and problem solving skills: In a container, there is 6845 l of oil, 5947 l more than in the barrel. How many liters of oil are in the barrel?
- Measurement and Geometry – Declarative knowledge: Are the following statements right or wrong? Explain your answer!

  - Every deltoid is square
  - Every square is a deltoid

- Measurement and Geometry – Procedural knowledge: What is the volume in cm^3 of a bottle of 50 dl?

- Measurement and Geometry – Strategic and problem solving skills: New carpeting of a room is planned. How long is the circumference when a room is 2 m 75 cm wide and 4 m 30 cm long? Make a drawing!

3. Results

Results of the pretest and the posttest are shown in Table 1 and 2 (results are given in percentage, because scoring of different parts may vary from part to part):

With respect to the results, although the tasks of the test cover the lower-level curriculum, there are still some items that do not reach 50%. The general comparison of the mean scores during the pretest and posttest reported in the two previous sections suggests that the mathematical method course had a significant and beneficial impact on the student teachers' competence in elementary mathematics. To summarize the results of the test it can be said that the knowledge of the teacher training's students is limited and uncertain. The results confirmed the frequently heard concern that at the beginning of their course students have rather weak mathematical competencies. At the end of their mathematical (methodological) course, the overall test performance had become substantially better, although there were still reasons to be seriously concerned about the readiness of some student teachers to teach mathematics to elementary school children.

Although not presented in terms of percentages, it is clear from the study, that those students who attended in the methodical course perform better in the posttest comparing with those ones whose institution has no specific methodical course. This very fact underlines the absolute necessity of methodical studies, not only from the pedagogical viewpoint, but also from scientific point of view - it seems to be an essential part of the curriculum to improve the knowledge of teacher training students in terms of basic mathematical notions and elementary strategic thinking. The authors think that every teacher training institutions must include methodical courses in their curriculum.

Unfortunately, the increasing of the number of students in higher education yields the consequence that more and more young people can be admitted from those ones who can not comply with the minimum requirements or can do it only in a very difficult way. This will effect on their work in the future where they will be uncertain and in worse case they will teach the next generation badly and faulty. As a result, the work in the lessons will be also irregularly which will be noticed by the students, too. The other source of the problem can probably be found in public education: students must learn a lot of material, but the world is changing and the attitude "I can get everything easily and only with little effort" does not

		Declarative	Procedural	Strategic
Numbers and Arithmetics	mean	61.0	63.3	47.8
	st.dev.	21.5	19.8	23.5
Measure and Geometry	mean	46.5	56.8	31.7
	st.dev.	24.1	29.0	28.2

Table 1: Pretest results (percentages) in terms of Declarative knowledge, Procedural knowledge and Strategic and problem solving skills in the two fields of mathematics

		Declarative	Procedural	Strategic
Numbers and Arithmetics	mean	70.1	68.8	60.2
	st.dev.	20.1	18.1	20.8
Measure and Geometry	mean	54.8	64.7	41.0
	st.dev.	20.9	25.3	26.4

Table 2: Posttest results (percentages) in terms of Declarative knowledge, Procedural knowledge and Strategic and problem solving skills in the two fields of mathematics

help the education of mathematics where you need precise knowledge of notions, strategies and a lot of practicing. Competence of mathematics is very important for the following generations. Problems from the real life can be used also in other sciences: the purpose and task of mathematics' teaching acquaints students with the concrete environment relations of quantitative and spatial circumstances that are surrounding them. Establishing their modern mathematical literacy therefore is of utmost importance, which makes them able to apply and develop mathematical thinking. Particular attention should be paid to the development and improvement of primary concepts, which should include various activities. Mathematics as a profession is to develop self-awareness of a starting experience, to improve independent thinking needs, to describe the joy of problem-solving and to develop positive personality traits. Some of the mathematical knowledge is abstract and a significant part is still connected to a specific knowledge. But emphasis should be placed on the diversity of activities to raise awareness of the experience, to record different ways, interpretation and systematization to search of correlations.

4. Discussion and concluding remarks

Starting from the state-of-the-art in the international research literature on preservice and in-service teachers' insufficient mastery in one of the major components of their domain-specific professional competence, namely their mastery of the content to be taught to their students, and its relationship with classroom practice, a longitudinal study was set up in which we assessed the elementary mathematical content knowledge and skills of a large group of Hungarian teacher training's students at the beginning and at the end of their methodical studies. Taking into account the Hungarian standards for elementary school mathematics, a pretest and a parallel posttest were constructed consisting of 27 items divided in six subtests, representing the major categories of these standards. Although none of the items required mathematical knowledge or skills beyond the content of the mathematics curriculum for the elementary school in Hungary, the test contained several items that demanded a thorough understanding of certain mathematical notions and/or the application of problem-solving strategies for using these mathematical notions in context problems. The results of the pretest confirmed the frequently heard concerns about the problematic level of mathematical competence of students who want to become an elementary school teacher given the low overall mean score as well as the detailed results for some very difficult items and for some very low performing subjects. The comparison of the actual mean pretest score and the score predicted by the teacher trainers indicated that Hungarian teacher trainers certainly do not underestimate the weakness of the mathematical content knowledge of their incoming students. Although the posttest results were considerably better than those for the pretest, the overall mean score was still very low. The design of the present study does not allow a more fine tuned analysis of the relative contribution of the instruction factor, and even leaves open the possibility that other factors besides this contributed to the observed gain in test scores between pretest and posttest. But we can definitely say that the methodological course help with the development of the students' mathematical knowledge and the correction of the wrong rooted concepts. There is an opportunity to the substitution of the missing knowledge, too. Meanwhile the methodological culture of the students is improving as well. The substantial differences in test score gain from pretest to posttest between the 5 institutes for teaching training that participated in the study suggest that these institutes were almost equally successful in developing the elementary mathematical competencies of their student teachers. But it is evident from the details of test results and answers, that students from those institutions, who provide separate methodical courses, can gain more well-established knowledge in explaining simple mathematical relationships and notions, than those students coming from institutions without separate methodical courses.

Additional research is needed to further unravel what characteristics of the teacher-training program in general, and of the specific mathematics (education) courses in particular are decisive for the development of the elementary mathematical competence of preservice teachers. Beside documenting the development of mathematical content knowledge and skills of preservice elementary school teachers in Hungary, the present study also resulted in two parallel versions of an instrument that is useful for the (self-) assessment of student teachers' mastery of the mathematical content they will have to teach after their graduation. The test as a whole proved to be a valuable instrument to assess the entrance level and the progress of mathematical content knowledge of our students, or to assist our student teachers in the self-assessment of (the development of) that level.

Currently, we are planning a follow-up study aimed at the development of a computer based instrument for continuous (self-)assessment of the mathematical content knowledge and skills of preservice teachers. Research evidence suggests that effective mathematics instruction involves the use of a variety of teaching methods. At the same time, there is general agreement that certain methods such as problem-based learning, investigation and contextualisation are particularly effective for raising achievement and improving students' attitudes toward mathematics.

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