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"How to solve it?" – The tsumego session*

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Abstract

The so-called 'Pólya's method' is now the canonical way of teaching mathematical problem solving. We would like to show that the method is not restricted to Math classes. Here we apply the method to solving tsumego problems that are isolated, small scale tactical problems in the ancient board game of Go. This new and unusual topic enables the students to get a wider view of the strategies of problem solving and the cognitive and psychological processes involved can also be easily demonstrated.

Keywords: problem solving, game of Go (Wei-Chi, Baduk), Pólya's method

MSC: 97A20,00A08,97D50

1. Introduction

Go is an ancient two player Asian board game with very simple rules (see Appendix A). Despite the simplicity of its description, the game is indeed very complex and requires deep strategical and tactical knowledge. In fact, Go is the last stronghold of natural intelligence, the last board game for which artificial intelligence up to now has failed to produce computer programs that can beat professional players. Go seems to require problem solving techniques that go beyond the brute force search algorithms and learning the game is rumored to be equivalent to take an advanced mathematical course. For younger people playing Go can improve thinking skills and rather surprisingly it can ease their social interactions[8] as well. Similar to chess problems there are Go problems called *tsumego*s (Japanese term adapted to English, see Appendix B). These can be introduced without explaining the

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full complexity of the game, so problem solving can be studied in a very focused setting, unlike mathematical problems that sometimes require some background knowledge.

Pólya's method described in his seminal book titled *How To Solve It?* [6] is now the standard way of teaching mathematical problem solving. The method distinguishes four principles or rather four consecutive stages of problem solving.

Understanding the problem Restating the problem in easier terms with more explanation, drawing diagrams, formulating questions, etc.

Devising a plan Assembling a list of possible steps leading to a solution, guessing and checking, considering special cases, eliminating possibilities, etc.

Carrying out the plan Executing the steps - patience and care is needed.

Looking back, evaluation The real in gain in the learning process comes from reflecting on what has been done and how.

These steps are general enough to apply them in a context different from mathematical problem solving. Here we describe a 90 minutes long session where students solve Go problems using Pólya's method, demonstrating how each step of the method applies to tsumego solving. This description is detailed therefore by using this description, similar sessions can be carried out in different environments. It is important to note that deep knowledge of the game is not required for the instructor.

2. The Tsumego Session

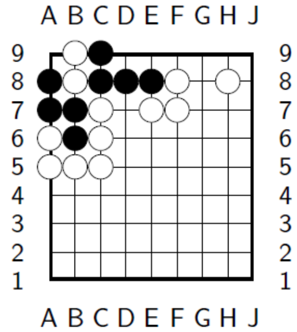
At the Eszterházy Károly College, as part of a one semester programme for fostering talented students from secondary schools we had afternoon sessions on different topics in Mathematics and Computer Science. The pupils were from different schools chosen by teachers from their schools, 12 pupils in total. This particular session on problem solving consisted of two parts (each of them 90 minutes long). The first part contained classical mathematical problems with explicit reflection on the heuristics. Due to the length of the session, the afternoon was very demanding for the students. Therefore it was very important for the second part to be more entertaining, even slightly unusual, thus we chose the game of Go.

It is necessary that real Go boards and Go stones are used during the session. Proper Go equipment has distinct aesthetics: simple regularity of the grid contrasted by the organic shape of woodgrains, the balance of interwoven black and white shapes. Invariably people start fiddling with Go stones when those are within reach of their hands, even when they are not in the situation of playing a game. Therefore the tactile experience of placing a stone on the board is very much part of the game. It is motivating and it gives a natural pace for working on the problems (as opposed to quickly clicking through all the empty intersections by a mouse while staring at a computer screen).

2.1. Understanding the Problem

Without further ado the students are presented with the following tsumego problem. (To save space in later diagrams we omit the coordinates.)

Problem 2.1. Black moves and lives in the corner while white is trying to kill the black groups.



Clearly, this immediate presentation of the problem will have a mild shock on the students as they are most probably used to long introductions before the first exercise. Obviously, this works better if the students have no prior knowledge of the game, or they just played a few games some time before, but they are not regular players. If Go players are present they should be asked not to spoil the effect by telling the solution quickly.

Using the confusion of the students the instructor can point to the first stage of problem solving: understanding the problem. In the ideal situation they have no prior knowledge of the game, so they have to face a situation in which understanding of the problem is completely missing. This never happens in mathematical problem solving, since by the time they first hear about Pólya's method, they already have solved many problems so their background knowledge is indeed quite deep.

Trying not cause any frustration by overexploiting the initial confusion the instructor claims that understanding the problem is just a matter of a few minutes long explanation. Unlike chess, where each piece has its own style of moving, go stones are all the same and once placed they do not move. Fortunately, for life and death problems, only a few concepts needs to be introduced. A *group* is a set of connected stones (along the lines, but not diagonally). A *liberty* of a group is an empty neighbouring intersection. A group is dead if there are no liberties left, so the number of liberties measures how far is the group from being captured (see details in the appendices). For unconditional life a group needs to have at least two liberties, two empty intersections that are not connected along the lines, i.e. they are separated by the group itself (see Fig. 1 and the Appendices).

The goal is now clear: to make moves in a way that the black group eventually survives by building a living shape or captures some white stones.

This introduction of the basic concepts of the game is a good opportunity for introducing the game in a wider philosophical[2] and historical context[3, 4]. In fact,

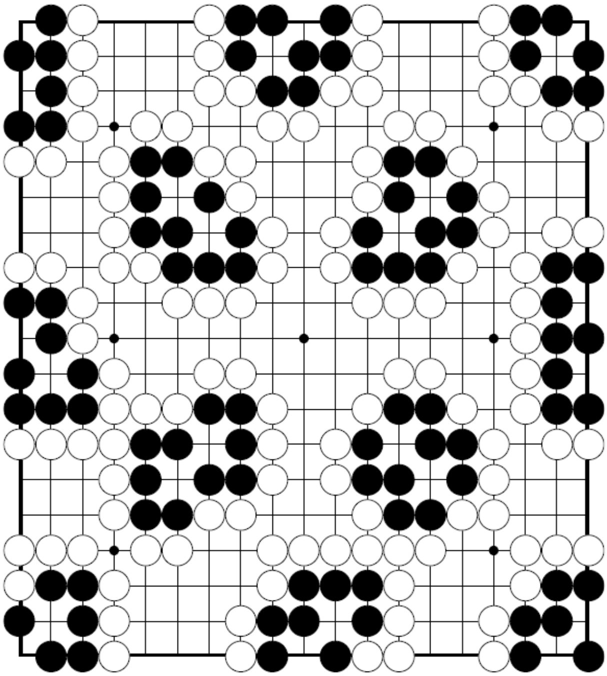


Figure 1: Minimal living shapes. Each black group has only two remaining liberties, but these liberties are well separated, therefore these groups cannot be captured, they are alive unconditionally. For the very first time the concept of unconditional life may not be fully comprehended by the students. This is not a problem, but it is still useful to show this collection as some students may recognize one of these shapes later on the board. [7]

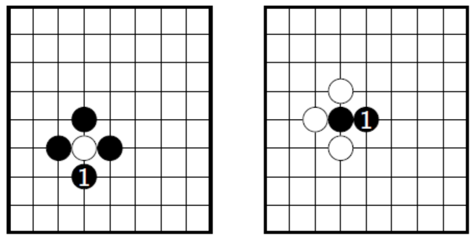


Figure 2: Problems for the concept of capturing and escaping by increasing the number of liberties. The key first step is indicated in both problems. For both problems the solution consists of only one move.

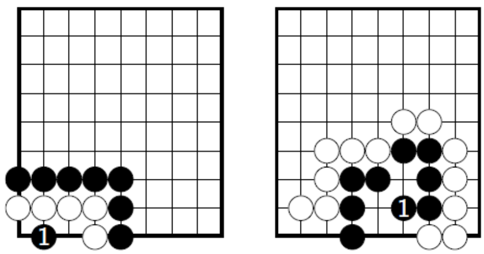


Figure 3: Problems for the idea of having two eyes. In the left problem the white group is not able to form two eyes (comb shape) after Black 1. In the problem on the right Black occupies the only point that can separate two empty points within the tentatively surrounded area.

we do nothing special here, only reversing the usual order of first the introduction and giving background information, then proceeding to exercises. Turning the order around is done for giving more motivation for the pupils and for illustrating more vividly the first step of Pólya’s method.

2.2. Devising a plan

By now it is clear for the students that the plan will consist of a sequence of alternating moves. But there are many possible choices and a complete beginner may not have a sense of direction to follow in solving the problem. Another advice from the problem solving method is that one should look for similar but simpler problems. For this purpose the students are given five simpler problems.

The first two problems in this set are just checking the understanding of the basic concepts: capturing and escaping by increasing the number of liberties of a group (Fig. 2). Interestingly, students found these problems too easy and difficult

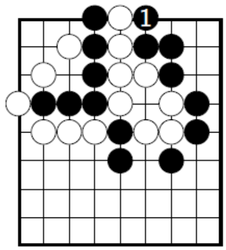


Figure 4: A capturing race. The solution of the problem requires to put white into atari at the right place. Trying to capture the other white group will end up loosing the inner black group.

to believe that the answer is just placing a stone. Therefore it is important to reiterate that now we look at simplest possible problems.

The next two are about the idea of unconditional life by making two eyes (Fig. 3), referring back to the collection of unconditionally alive shapes (Fig. 1).

The last one in the set is a capturing race (Fig. 4) where for solving the problem one has to count the liberties of each group involved. In all problems black is to make a move. This is just a convenient simplification.

2.3. Carrying out the plan

The best setup is when students work in pairs on one board, one of them taking black, the other one white. After an unsuccessful attempt they may swap sides. If they cannot come up with the correct solution the instructor can take black (or white) and play it out with the students.

During the session approximately one third of the student came up with the correct solution without any further instruction. Others needed feedback on evaluating actual positions, whether the goal is reached or not.

At this point it is good to show the tree structure encoding the variants of a tsumego problem as an illustration. The nodes of the tree are positions, the connecting edges are moves. The variations can be studied after trying to solve the problem on an excellent tsumego site [5]. This also enables a quick explanation of the basic idea of the classical artificial intelligence algorithms: searching the game tree [1]. Humans do exactly the same type step-by-step calculation in an unfamiliar situation just as the participating students did during the session.

2.4. Reviewing the Solution

It is a good attitude in Go if someone is looking for a better move even if a good move has already been found. After successfully solving the tsumego it is important to evaluate the solution. Is there a better variant? Did we overlook something? Maybe white can intervene at a certain move? In case the solution is solid, still there is room for improvement. One can consider whether another shape would be better for further development.

It is also important to mention that the applied search method is not the highest form of problem solving. There is empirical evidence that Go masters come up with the solutions without any discursive thinking, their eyes fixate on the vital point under 300 milliseconds [9]. Some sort high-level pattern matching is done by the master players. For beginners the eye movement traces the steps of the search method. By solving tsumegos the brain develops this ability to recognize patterns on a subconscious level. This level of problem solving is in contrast with the discursive search method mentioned before. Instead of thinking about the problem and creating a plan, we can simply “see” the solution. Clearly the level of intuitive knowledge, the immediate certainty can be reached only by frequent practice of the step-by-step problem solving. Most players agree that this is the best way to get stronger in Go, solving many tsumegos frequently.

Similar is true for mathematical problem solving. By working on many different problems one develops the expertise or rather the intuition to see parts of the solution even in more complex problems. This is a well known phenomenon for working mathematicians. When working on a problem the solution comes suddenly, and not when someone tries hard, but when turns away from the problem. This is the next level beyond problem solving as an exercise towards creative research.

Clearly, the success of the session depends on whether the students were capable of solving the tsumegos or not, but since there are really elementary problems, this can be guarateed. The instructor should emphasize that a lot has been learnt and with the acquired knowledge the students can start to play the game themselves. They should be provided with further technical information (good starting point is [7]). At the same time the students should be warned that this is just the beginning and becoming a good Go player or a good at Math is not a quick process.

3. Conclusion

We described the application of Pólya’s method to a different domain of problem solving in a form of a special session for selected students. This enables students to get a new perspective on the steps of problem solving (see comparison on Fig. 5). Solving Go problems is a great opportunity to talk about the psychology of problem solving, to introduce algorithmic concepts of artificial intelligence and the inner workings of our pattern matching minds. This fresh view of Pólya’s method helps students to apply the steps more efficiently in mathematical context as well. We recommend the tsumego session as a complement to traditional problem solving classes.

	Mathematical Problem Solving	**Studying Tsumegos**
Previous knowledge	Requires extensive background knowledge and experience in Mathematics.	Nothing needed. Only a few simple concepts are to be explained.
Benefit	Good preparation for tests and exams.	Fun, gives new perspective on problem solving, but no immediate payoff.
Reflection	Doing Mathematics is very complex activity, an interplay of numerous cognitive structures.	Due to its simplicity it is easy to point out the cognitive processes involved.

Figure 5: Comparing a mathematical and a tsumego solving session.

A. The Rules of the Game of Go

Go is played by two persons (Black and White) on a board with a 19×19 grid. The game starts with an empty board. A move is placing a stone on an empty intersection point. Black makes the first, then moves alternate. The goal is to surround more territory. Friendly stones on neighbouring points (connected by gridlines but not diagonally) form *groups*. By counting a group's empty neighbouring points we get the *liberty* of the group. If this number becomes zero, i.e. all neighbouring points are occupied by enemy stones, then the group is captured or dead and it is taken off the board. If the liberty count is exactly 1, then we say that the group is in *atari*. It is not compulsory to make a move but suicide moves and those that restore a previous board position are forbidden. The game ends when both player pass. Then the surrounded territory is counted (the number of prisoners subtracted). The winner is the player with more territory.

B. Tsumego

Tsumegos are local battles where in a few moves one side suffers decisive loss or gains overwhelming advantage. The most common type of these all or nothing situations are *life and death* problems in which the goal is to save or kill a group, i.e. increasing liberties/forming two eye groups or filling up liberties of enemy groups. Solving tsumegos is basically finding a few key moves. Go players aim to solve tsumegos within seconds.

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References

[1] E.R. Berlekamp, J.H. Conway, and R.K. Guy. *Winning ways for your mathematical plays Volume 1*. A.K. Peters, 2001.

[2] William Cobb. *Reflections on the Game of Go: The Empty Board 1994-2004*. Slate and Shell, 2005.

[3] Michael H. Koulen. *Go. Die Mitte des Himmels - Geschichte, Philosophie, Spielregeln, Meisterpartien*. Hebsacker Verlag, 2006.

[4] Shirakawa Masayoshi. *A Journey in Search of the Origins of Go*. Yutopian Enterprises, 2005.

[5] Adam Miller. Community site for solving go problems. `http://goproblems.comgoproblems.com`, 2011.

[6] George Pólya. *How To Solve It*. Princeton University Press, 1945.

[7] Comprehensive wiki page on the game of go. `http://senseis.xmp.net/senseis.xmp.net`, 2000-2011.

[8] Yasutoshi Yasuda. *Go as Communication: The Educational and Therapeutic Value of Go*. Slate and Shell, 2002.

[9] Atsushi Yoshikawa and Yasuki Saito. Perception in tsumego under 4 seconds time pressure. In *Proceedings of the eighteenth annual conference of the Cognitive Science Society*, page 868, 1996.