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Mathematical competences examined on secondary school students

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Dedicated to professor Béla Pelle on his 80th birthday

Abstract

First I'd like to write about the idea of mathematical competence and make clear its components, according to the 2000 Lisbon Resolution of the European Union. That, and the results of the first PISA examination implicated a development work in our country, and as a result we now have new types of competence-based school-books, and the methodological culture of teachers is under reformation too. But these changes are not uniformly welcome among teachers. There is a kind of contradiction between traditional and competence-based education and evaluation – so I tried to match the two types with measuring the present level of competence of secondary school students, and upon that work out development methods, in which the defects of the knowledge can be supplied.

The students solved an exercise-paper with 5 exercises (practical, playful, needs many different abilities and skills, but less knowledge, both simple and complex) in 45 minutes. I made two types of measurements – a traditionally used method which measures the mathematical achievement, and another, which measures the level of skills and competences. I analysed the results with comparing the achievements with the competences to show how the mathematical skills prevail among achievements. I show this analysis with graphs in my paper. According to the results of the measurement, we can declare the main areas of development. At the end of my paper I'd like to show how I plan the follow-up of this examination.

1. Introduction

Nowadays we can hear a lot about different competences and competence-based teaching. The European Union in 2000 Lisbon Declaration gave recommendation to its member states to develop their educational system. Moreover the results of international measurements on the knowledge of Hungarian students also required reformation. According to the measurements, it seems that our students' knowledge lags behind the knowledge of other countries', it isn't modern enough for the challenges of present days. On these effects new projects were realized to develop education in our country too. In 2002 within the confines of the National Development Plan the key competences were defined, which the education has to take attention on. Through the Human Resource Development Operative Programme started curricular and methodological generative works, and after that the tests of the developed programmes in schools through tenders. Now we have tangible results of it in school-books and other learning materials, programme packs. But the spread of using these new methods is delayed by the doubtfulness of teachers, sometimes negative behaviour, the slothfulness of the changes of educational procedures, and also the negative reactions of parents. It's hard to accept every newness, but it's easier if well prepared and supported by examinations. It's a pity, that it was not like this in Hungary, so that's why we have this resistance against these changes.

The aim of my assessment is to see the appliable mathematical knowledge of secondary school students learning according to traditional curriculum with a pretest. After the evaluation of the test – comparing competences and mathematical achievement too – I would like to look for development strategies and methods, which are valid for the general school system to appraise mathematical competences within the maths lessons, and not in extraordinary time.

2. Mathematical competences – theoretical considerations

The key competences are indispensable for the flexible accomodation to changes, the acceptation of changes and the forming of own life. Mathematical competences belong to them. The OECD in connection with the PISA-examinations drew up the competence like this in 2000: "Mathematical competence is a preparedness, which qualifies the person to identify mathematical problems, understand and handle them, and also to form a valid opinion on the act of mathematics in present and future professional and private life, and the role in personal and social connections." [2]. According to this the following components were defined:

1. mathematical thought, conclusion
2. mathematical argument, proof
3. mathematical communication

4. mathematical modelling
5. problem posing and solving
6. representation
7. symbolic and formal language, operations
8. tool using skills.

These components can be on three development levels:

a. reproductive – executing routine, standard exercises, using definitions,

b. connective – executing complex, but still standard exercises, using integration,

c. reflective – handling complex problems, genuine approach, generalization.

Psychologists and theoreticians in pedagogy devided these components to more skills and abilities according to factor and matter analysing [2]. Let's see such a division:

Skills	Thinking abilities	Communication abilities	Cognitive abilities	Learning abilities
numbering, counting, quantitative conclusion, estimation, measuring, changing units, solving textual exercises	systematisation, combinativity, deductive conclusion, inductive conclusion, probability conclusion, argument, proof	relation vocabulary, textual understanding and explanation, spatial sight, perception of spatial relations, representation, presentation	problem sensitivity, problem representation, originality, creativity, problem solving, metacognition	listening, perception of parts-whole, memory, exercise-keeping attitude, exercise solving velocity

If we think it over, these components are really important parts of the competitive mathematical knowledge, but they mean so multiple and diverse knowledge system, that even some mathematics teachers don't dispose all of these skills and abilities on the needed level. Such a division could hardly be used in everyday school-life during maths lessons, because of its complexity.

In the 2006 PISA report we can find another desciption of mathematical competence: "The applied mathematical literacy means, that the person recognizes and understands the act of mathematics in real world, forms valid decisions and his/her mathematical knowledge helps in solving own life's real problems, and become a constructive, enquirer, moderate member of the society." [2]. This description is more general, then the previous.

If we look after in different sources, we can find many different definitions. This multiple approach shows that the research of the theme is not closed, its special literature is under evolution, and the educational methods based on it are under developing yet. On the other hand, it's hard to accept such a diversity. If we lay down a few (6-8) guiding principles to go along during a development programme, it's easier to follow. That's why I use the above mentioned 8 points with the 3 levels to define mathematical competences, as it is in English usage. These points draw up handable abilities, don't touch on different parts of mathematical literacy, general enough to cover the whole spectrum of mathematics, and all the skills and abilities mentioned in the tablet could be classified under one of the 8 points.

3. Traditional education – competence based education

Before we compare the traditional and competence based education we have to make clear, why we teach mathematics. After defining and deciding the goals can the materials and methods be determined. The materials of education are assigned by social expectations, we have to cultivate such skills and abilities, that are essential to social integration and forming own life. And of course, in working out materials and methods pedagogical and psychological wiewpoints must be taken into consideration too [3]. The acceleration of changes in society and technology gives new challenges to the members of education, and if we react too late, we'll lag behind. After the weak results of the first PISA assessment in 2000, Germany started a development in education, accepted and supported by wide rates of the society, which caused a significant growth in the 2003 and 2006 results [2]. We'd need a similar quick development too. In teaching mathematics we can develop many psychic attributes, that are important parts of social integration and lifestyle. This is one of the most important pedagogical goals. As an educational goal, we can declare to give such experiences, images, ideas and knowledge during teaching mathematics which works up the ability of simple and complex using of abstractions, connections, terminology, operations, cognitive actions. The most important qualificational goal is to make the students able to use their knowledge creatively, and work up routines and such personal abilities that help them in lifelong learning process [3].

I think, that the traditional, matter-concentrated teaching has got many valuable moments, which shouldn't be dropped out, but should be saved among new forms and methods. If we look into old school-books and exercise-collections, we can find many practical textual exercises, that could be used as nowadays as 20-30-50 years ago, only we have to make the text current sometimes. Our secondary school mathematics curriculum aspired to give knowledge from many different parts of mathematics. The revision of the National Basis Curriculum and the frame curriculum in 2003 dropped out such parts, that were articular in maths curriculum and maturity earlier, but the needth was querying (e.g. trigonometric equations, additional theorems, etc.) These changes showed the signs of modernisation, and also some new parts were put in, such as statistical counts. These changes and emphasis shifts to new themes are the results of the 2000 Lisbon Declaration, which started the reforms [7]. I think, the material is still too wide, so that the teachers don't have enough time for practising and problem solving. The decrease of number of lessons, the increase of number of students in one class, and the traditional circumstances of teaching frames hardly give enough space to use new methods efficiently. The structural changes in secondary school system from the 1990's on also makes the efficient work hard: though the number of children weakens year by year, more students learn in grammar schools now, where traditionally more theoretical knowledge is expected then in the previous practical, vocational training schools. So, many of those children has to learn theoretical knowledge who don't really need and claim it – because the practical schools disappeared or changed. This also raise the question of restructuring the teaching materials and the learning methods. But to achieve a real and efficient competence based education we have to change the frames of school timing too.

4. The participants and the test problems of the assessment – research question

The aim of my assessment is to see the appliable mathematical knowledge of secondary school sudents learning according to traditional curriculum. This is a pretest of a development strategy on mathematical competences (mainly modelling and problem solving). How can we simply assess the level of abilities together with the mathematical achievement? We used to assess only the achievement in school, but for such a development work I'd need a combined evaluation of the two things. I made my assessment in the Practising Secondary School of Eszterházy Károly College in Eger, in 12 grammar classes (3 classes per grade) with test papers laboured for 45 minutes. Altogether 278 students did the tests (77 on 9th grade, 48 on 10th grade, 82 on 11th grade, 71 on 12th grade). These grammar classes learn standard mathematical curriculum, 3 lessons per week, no special maths classes among them. The main profile of the school gives high level education to students in drawing and visual communication, music, foreign languages, and information technology, with more lessons per week in these subjects than the average. So in mathematics they have got average preparedness. At least half of the students come from other places, many of them live in student hostel, and the rate is even higher among the special classes. The students didn't get special preparation for this assessment. They solved the exercises during a normal maths lesson, and I would like to thank to my colleges, Mrs. Győzőné Erdős, Mrs. Zoltánné Pelbárt and Mrs. Katalin Lénártné Pintér for their helpfulness.

I got the test problems from competence based school-books made by Educatio Kht., partly from "Secondary school mathematical exercise collection" from National School-book Publisher, and also from the collection of the MA students of "teacher of Mathematics" in Eszterházy Károly College. When collecting and selecting the exercises I took into consideration, that they should:

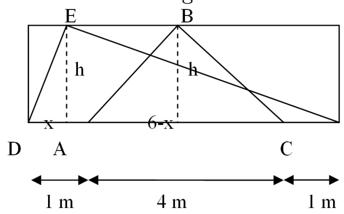
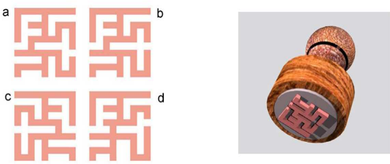
- be practical,

- measure varies of skills and abilities,
- need only such knowledge, that the students already learnt in previous school years,
- be playful, diversified,
- be both simple and complex exercises among them.

I gave 5 different exercises to every grade, which the students had to solve in 45 minutes. The time was far enough for those ones, who read slowly too. The exercises were all textual, so the weakness of reading and understanding ability naturally influenced the results. But, unfortunately, it is not enough to make only one test, to find out, how this weakness effects on mathematical achievement, just during continuous work with a student – so I couldn't take this into account. Every papers had exercises to measure counting skill, logical thinking skill, combinativity, functional, algorithmical thinking, spatial sight, perception of spatial relations, problem solving abilities. In assessing the exercises I took into consideration which thinking or counting units would lead to the result (but of course, different ways could be right) [1]. I made two types of assessment to each papers: a mathematical achievement evaluation, traditionally used in school practise by teachers, and a new type of skills – ability level assessment [4]. I would like to choose the main path of development strategy by comparing these two, in order to develop skills and abilities of students, so as to result growth in mathematical achievement in school too. An example of the test (made for the 11th grade) and the two types of assessments can be seen below. The paper of other grades contains similar exercises.

Table 1: Exercise paper for 11th grade

<table>
<tr><th>Exercise</th><th>Solution and achievement points</th><th>Competence points</th></tr>
<tr><td>1. Five friends noticed, that their telephone numbers are such 7-digit numbers, which's first digit is 3, every digit is different, and the buttons on the mobile phone pushed one after the other goes in the order of the move of horse in chess. Which are the five numbers? Can there be more numbers like these?
<table>
<tr><td>1</td><td>2</td><td>3</td></tr>
<tr><td>4</td><td>5</td><td>6</td></tr>
<tr><td>7</td><td>8</td><td>9</td></tr>
<tr><td></td><td>0</td><td></td></tr>
</table></td><td>Possibilities according to the conditions:
3-4-9-2-7-6-0 (1p.)
3-4-9-2-7-6-1 (1p.)
3-4-0-6-7-2-9 (1p.)
3-8-1-6-7-2-9 (1p.)
3-8-1-6-0-4-9 (1p.)
There can't be more, because keeping the rule of chess move, the numbers would repeat.
Argument: tree graph. (1p.)</td><td>a,
b,
c,
g,
h,
i,
j,
k,
m</td></tr>
</table>

2. Feri's father would like to hang two 90° angled halogene lamps in their cellar. One into the middle of the 6 m long ceiling, lighting straight down, but in this case 1-1 m long stripes would left unlighted. The other lamp obliquely in another place, to lighten the whole floor. How far is the second lamp from the middle? See the figure below!  	Interpretation, notation on figure: (2p.) $ABC_\triangle$ isosceles, rectangular, so the altitude $h = AC/2 = 2\,\text{m}$ (2p.) $DEF_\triangle$ rectangular, the altitude belonging to DF hypotenuse is also 2 m, the two parts of the hypotenuse are: x and $6 - x$ long. (2p.) According to altitude theorem: $x(6 - x) = 2^2$ (2p.), from this we get: $x^2 - 6x + 4 = 0$. Using the solution formule, we get $x_1 = 3 - \sqrt{5} \cong 0.76$ and $x_2 = 3 + \sqrt{5} \cong 5.24$. (2p.) Conclusion: he has to put the second lamp (E) in $3 - 0.76 = 2.24\,\text{m}$ distance from the first one (B). (2p.)	a, b, c, f, g, h, i, k, l, m
3. One day Barbara, Bea, Bori and Balázs travelled by train with their friends, and for passing time, they played. At first every member of the company had to think of a 3-digit positive number, which's digits are bigger then 4 and less then 7. When they told their numbers one by one, they realized, that all numbers were different. a) How many were they at most? An other day Barbara, Bea, Bori, Balázs and their 4 friends (Attila, András, Ali and Anna) went to cinema together. All the 8 places, on the tickets, were in one row, next to each other. b) In how many different orders can the 8 friends sit, if non of those, who's name begins with the same letter, can sit near each other?	a) The digits can be 5 or 6, and can be repeated. (1p.) The number of variations of 2 digits, in 3 places are $2^3 = 8$. (555, 556, 565, 566, 655, 656, 665, 666) So the company's got 8 members at most. (2p.) b) The sitting order can be ABABABAB or BABABABA patterned. (1p.) The number of orders with letter "A" are 4! altogether, and with letter "B" the same. (1p.) All the orders can be the multiplication of these: $2 \cdot 4! \cdot 4! = 1152$. (2p.)	a, b, c, d, e, f, g, h, i, j, k, l, m
4. Choose which figure is the imprint of the postmark on the picture!  	The "b" is the right imprint. (3p.)	a, b, c, i, m

5. A 130 m long freight train goes 42 kilometres per hour. What time does it go through a 220 m long tunnel?	The length of the train and the tunnel together is: 130 m + 220 m = 350 m = 0.35 km. That's the way to go if it wants to go through the tunnel. (2p.) According to relation between the way (s), time (t) and velocity (v): $t = s/v = 0.35/42 = 0.00833\,\mathrm{h} = 0.5$ min. So it takes the train 0,5 minutes to go through the tunnel. (3p.)	a, b, c, d, e, f, g, h, i, j, k, m

5. The assessment of mathematical achievement

I evaluated the completed papers with giving 1-1 points to each thinking or counting units, similar to the pointing system of maths maturity. I put the points per student and per exercise into an Excel tablet. I used the Excel to summarize the points of students, giving the achievement in percentage too, the average and deviation of each exercises, and each grades. I made graphs from the points of students, compared with the average, the minimal and optimal level. It caused difficulties that some of the students didn't get the test serious enough, because it didn't have any "stake" – it shows, that in our schools students used to work for marks, not really for knowledge. I tried to strain off these papers, because they wouldn't show the real knowledge and abilities, just the motivation (which is an interesting topic too, but it wasn't the main in my examination).

I appointed the minimal level of mathematical achievement at 20%, and the optimal level above 60%. Here are the summarized graphs of each grades. They show the points of the students in growing order, the minimal level, the average of the grade and the optimal level. Below the graph the tablets show the evaluation of each exercises per grade.

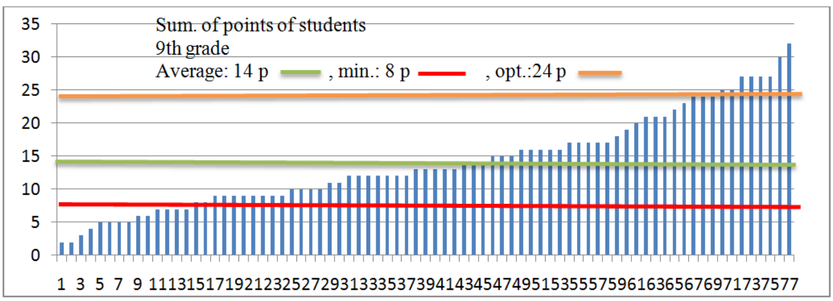


Figure 1: Assessment of achievement, 9th grade

Exercise	1. (8p.)	2. (12p.)	3. (12p.)	4. (3p.)	5. (5p.)	$\sum$ (40p.)	%
Average	3.30	4.71	3.75	1.44	0.81	14.01	35.03
Deviation	2.79	3.84	3.03	1.51	1.34	7.05	17.63

Table 2: Average of points and deviation on 9th grade

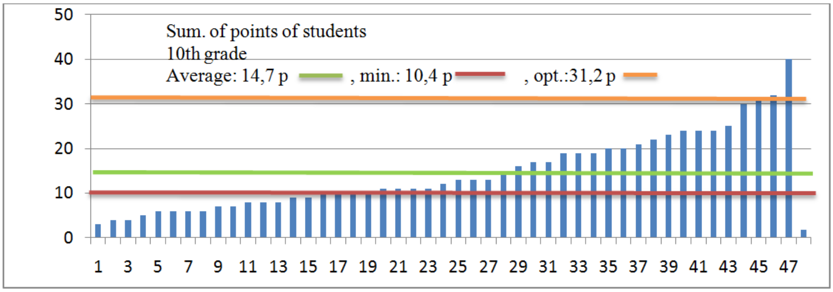


Figure 2: Assessment of achievement, 10th grade

Exercise	1. (6p.)	2. (12p.)	3. (10p.)	4. (12p.)	5. (12p.)	$\sum$(52p.)	%
Average	5.2	1.7	2.3	4.0	1.4	14.7	28.2
Deviation	1.69	2.85	3.61	4.47	2.43	8.51	16.36

Table 3: Average of points and deviation on 10th grade

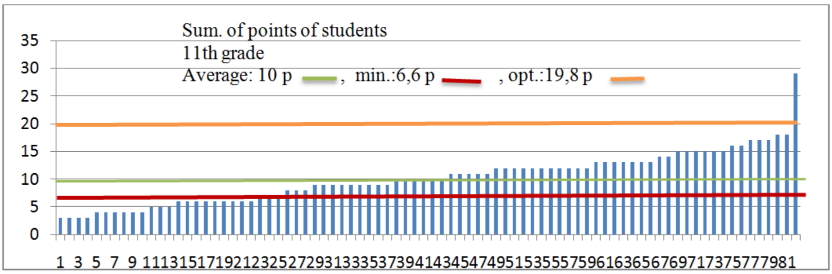


Figure 3: Assessment of achievement, 11th grade

Exercise	1. (6p.)	2. (12p.)	3. (7p.)	4. (3p.)	5. (5p.)	$\sum$ (33p.)	%
Average	3.39	0.56	2.98	1.76	1.50	10.18	30.86
Deviation	2.37	1.51	2.27	1.49	1.97	4.55	13.79

Table 4: Average of points and deviation on 11th grade

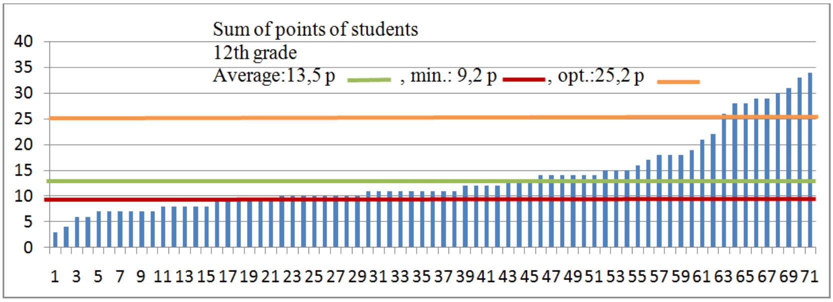


Figure 4: Assessment of achievement, 12th grade

Exercise	1. (4p.)	2. (12p.)	3. (8p.)	4. (6p.)	5. (16p.)	$\sum$ (46p.)	%
Average	3.45	1.18	3.44	2.23	3.32	13.62	29.61
Deviation	1.01	1.45	2.71	1.31	4.77	7.26	15.77

Table 5: Average of points and deviation on 12th grade

On every grade the students showed weak average achievement. The best average was on 9th grade, and the least on 10th grade (see tablet 2 and 3). If we look at the graphs (figures 1–4), we can see, that the averages of all grades are closer to the minimal level, than to the optimal. I could evaluate the work of 278 students, so we can consider the sample is representative for a normal grammar school, learning mathematics according to general curriculum. Altogether 74 students got points under the minimal level, that's the 26.5% of all. Their school achievement is rather weak too, we can declare. Above the optimal level were only 23 students, that is the 8.3% of the sample (see figures 1–4). I think one reason for the weak achievement is, that these exercises were unconventional, strange for most of them, and they weren't prepared for the test. On the other hand, I think the lack of motivation also weakened the results.

In general we can say, that the playfully interpreted, simple exercises, which didn't need too much counting, were more successful, than the complex ones. The result of those exercises, needed only spatial sight were much better, then those ones in which they had to count something from a given figure. The result of the exercises needed logical thinking and combinativity were better then the average as well. The weakest results were the complex exercises, which needed problem solving abilities, planning and more relations (see tables 2–5). I think, in this case the difficulty was the translation of the real problem into the language of mathematics. In my opinion, another reason of this weak result is a kind of laziness, because this generation prefers the easily, quickly available results to the ones demands patience and endurance. I think this problem is more related on present social problems than mathematics teaching, but in school we have to take this also into consideration, and it can be developed with traditional methods as well as competence based methods.

6. The assessment of skills and abilities

Still we rarery meet the measuring of skills and abilities in public education. After the first PISA assessment, started the Country Competence Measurement, which measures the abilities of students in textual understanding and mathematical tool usage. The system of this measurement shows into good ways of developing, because it measures the same grades (4th, 6th, 8th, 10th) year by year, among the same circumstances. It also followed by a social survey too. From this, we can follow up the development of these abilities of children year by year, and the performance of schools too. The problem is, that most of the students, parents, and even much of the teachers don't know, what this measurement is really for, and what the results really mean. It is hard to compare the ability points to the traditional school marks. That's why I tried to make a kind of comparison of the two types of evaluations. Because whatever we say, the achievement (marks) is important in school life (and further too), and we would like to see, how skills and abilities come out in achievement.

For the above mentioned purposes I wanted to examine the skills and abilities, so I made another kind of assessment, according to dr. István Czeglédy, used in a survey in Miskolc, among elementary school students. [4] I identified which of the following items can appear in the solution of each exercises (in a more complex one, all of them, in a simple, some), and gave simply 1 or 0 points, if an item appeared or not:

a) does he/she begins the exercise?

b) does he/she interprets the exercise well?

c) is there any valuable in his/her work?

d) does he/she make figure, tablet, systematize data?

e) is the figure, tablet, systematization valid for the solution?

f) does he/she use notations?

g) does he/she make plan?

h) is his/her solution purposive (even if there was no written plan)?

i) is he/she motivated to solve the exercise?

j) does he/she explain statements?

k) does he/she look for causal connections, relationships?

l) does he/she try for visualize in short forms?

m) does he/she try for whole solution, give full answer?

The competence points identified for the exercises are the following:

	1st exercise	2nd exercise	3rd exercise	4th exercise	5th exercise
9th grade	Balance of scales	Eggs for Easter	Combinatory	Imprint	Train
Altogether: 54 points	a,b,c,d,e,i,j, k,m	a,b,c,d,e,f,g, h,i,j,k,l,m	a,b,c,d,e,f,g, h,i,j,k,l,m	a,b,c,i,m	a,b,c,d,e,f,g, h,i,j,k,m
10th grade	Spinning dice	Selling shirts	How old is the captain?	PIN-code (comb.)	Place of the well
Altogether: 50 points	a,b,c,i,m	a,b,c,f,g,h,i, j,k,l,m	a,b,c,g,h,i,j, k,l,m	a,b,c,f,g,h,i, j,k,l,m	a,b,c,d,e,f,g, h,i,j,k,l,m
11th grade	Phone numbers	Lamps in the cellar	Combinatory	Imprint	Train in tunnel
Altogether: 49 points	a,b,c,g,h,i,j, k,m	a,b,c,f,g,h,i, k,l,m	a,b,c,d,e,f,g, h,i,j,k,l,m	a,b,c,i,m	a,b,c,d,e,f,g, h,i,j,k,m
12th grade	Filling dishes	Taxing in Zed	Cost of the horse	Queer money	Tangential trapezoid
Altogether: 54 points	a,b,c,i,k,m	a,b,c,f,g,h,i, j,k,l,m	a,b,c,d,e,f,g, h,i,j,k,l,m	a,b,c,d,e,g, h,i,j,k,m	a,b,c,d,e,f,g, h,i,j,k,l,m

Table 6: Competence points to each exercises

Evaluating the papers according to these viewpoints I summarized the competence points of all students per grade in an Excel tablet. The next step was to compare the achievement and competence points of students. How could I show, which students achieved above, according to, or below the level of their competences? For examining this I took the quotient of the students' competence points (C) and achievement points (A). My hypothesis was, that there can be a special relation between the two kind of assessments, and I tried to show and analyze it. I adjusted the students (per grades) into order of growing achievement points, and represented the C/A quotient on graphs. On the graphs we can see how many students did the test per grades, and the value of the C/A. I identified the "ideal" value of the quotient (maximal achievement points/maximal competence points) for every grade, the average and deviation of grades too (see figures 5–8). The graphs show interesting coherence.

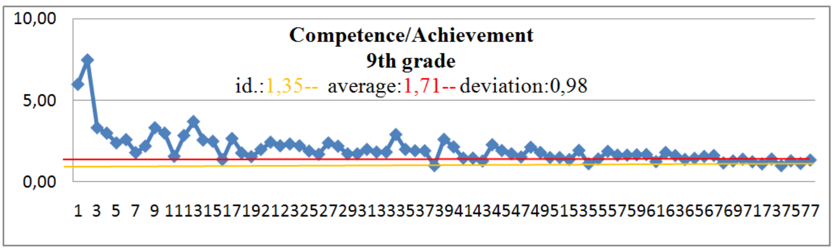


Figure 5: C/A in order of growing mathematical achievement, 9th grade

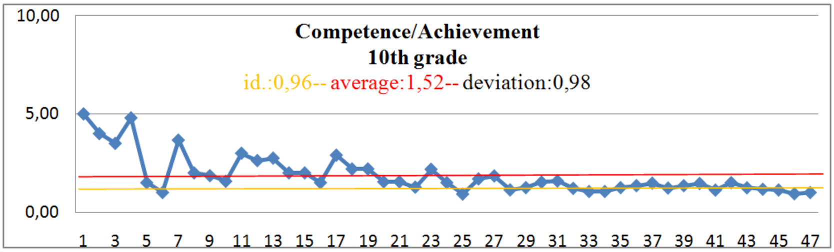


Figure 6: C/A in order of growing mathematical achievement, 10th grade

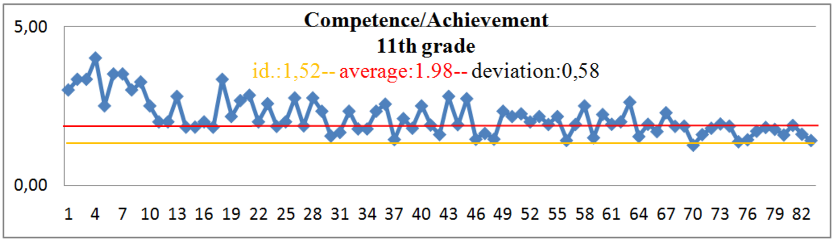


Figure 7: C/A in order of growing mathematical achievement, 11th grade

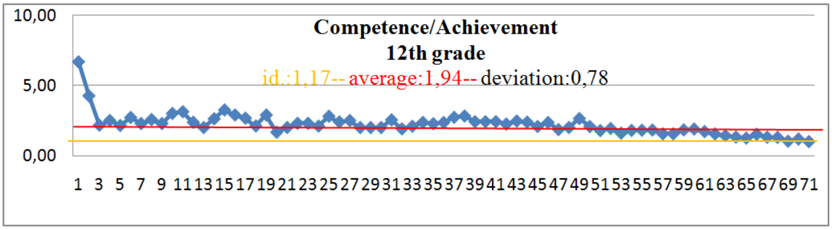


Figure 8: C/A in order of growing mathematical achievement, 12th grade

The graphs show the ideal level of C/A with orange line, and the average of the grade with red line (see figures 5–8). How can the C/A quotient be interpreted? It means that more the level of the quotient is above the ideal line, the student's mathematical achievement is more below his/her competences. If it is close to the ideal value, it shows, the student's achievement adequates to his/her competences. And if it is below the ideal line, the student's achievement is beyond his/her competences.

On every grades the graphs show an exponential-like degrease. This means that if one's achievement is better, his/her competences come up better in achievement. This also shows, that worse achievement doesn't always mean worse skills and abilities, but other factors too: lack of motivation, deficiency of previous knowledge, reading or understanding problems, etc. Mapping this needs a longer examination.

But such an analysis would help the maths teachers a lot in evaluating how the students can show up their mathematical competences in solving exercises. On the other hand it would help to decide which way to develop the students: some skills, abilities are weak, or the knowledge. We can see students achieving above and below the competences on every achievement levels, but if we look at the right end of the graphs, (figures 5–8) we can see, that those students, whose achievement points were better, all "brought out" their skills, abilities. In all, more students achieved below his/her competences, than above (see table 7).

Achievement	*below competences*	*adequate to competences*	*above competences*
9th grade	43 students (56%)	22 students (28%)	12 students (16%)
10th grade	26 students (55%)	19 students (40%)	2 students (5%)
11th grade	55 students (67%)	23 students (27%)	5 students (6%)
12th grade	56 students (79%)	12 students (17%)	3 students (4%)

Table 7: Number of students and percentage of the level of C/A

These numbers show that the main problem is, that for much students their skills and abilities don't come up in solving exercises as mathematical achievement – this could be developed by thorough knowledge, well structured and deliberate matters. I think, we'd need less, but in practical problems better appliable knowledge in our maths education. Nowadays after acquiring new matters (70–75% of all lessons) we have little time, only the 15–20% of the lessons, to practice and deepen the knowledge. I think, we'd have to change this rate into the growth of practising to get better results in problem solving achievements.

7. Further examinations, working out development methods

Beyond the examination of the present level of skills and abilities the statistical analysis of competence points is also helpful to find out which of them accured the least during solving the exercises. From the summary of the competence points I pried those points out, which appeared in less than one third of the students' solutions – I think these are the competences principally needs to be developed. There are some exercises, which were solved by more than one third of the students (4th exercise on 9th grade, the first exercise on 10th grade, the 4th exercise on 11th grade and the first exercise on 12th grade), so I didn't mention them – these were short, mainly "choosing from given answers"-type exercises. The next tablet shows the weaknesses (table 8):

9th grade	1st ex.: d, e, k, m 2nd ex.: f, g, h, j, l, m 3rd ex.: f, g, h, j, l, m 5th ex.: d, e, f, g, h, j, l, m	11th grade	1st ex.: g, j, 2nd ex.: b, c, f, g, h, i, k, l, m 3rd ex.: f, g, j, l 5th ex.: d, e, g, h, j, l, m
10th grade	2nd ex.:h, j, l, m 3rd ex.:g, h, i, j, l, m 4th ex.: f, g, h, j, l 5th ex.: f, g, h, i, j, k, l, m	12th grade	2nd ex.: f, g, h, i, j, l, m 3rd ex.: d, e, f, j 4th ex.: d, e, g, h, j, m 5th ex.: g, h, i, j, l, m

Table 8: The worst competence points in exercises

The above statistics show that most defects are

- in planning (g),
- in purposive solution (h),
- in explaining statements (j),
- in visualizing results in short forms (l),
- in trying to give full answer (m).

The present examination was the first step of a longer research, and would be followed by further comparing assessments according to my aims. The results show that we should start development on the following topics:

- “translating” exercises, problems from vernacular words to mathematical symbols,
- working out typical exercises, patterns for using mathematical tools,
- planning solutions of complex exercises, and working out solutions,
- developing exercise keeping ability, patience, extended attention,
- developing argumental and proofing abilities.

The next step of my research is to work out exercise papers on the above mentioned topics, which are appliable for a 45 minutes long lessons and a whole class of 30-35 students. In point of methods, I think, both cooperative and individual learning forms have got their own place in learning mathematics, so as in development work too. My opinion is, that variable usage of different methods could be the most efficient, because none of these forms result increase on its own – we have to find the right rates. The most difficult is to develop those psychical abilities, which are needed for long lasting concentrations, and dividing a complex exercise to parts. But these abilities are essential to social integration and lifelong learning and development. This also poses the question of too much materials – we'd need less, to get time for deepen knowledge. It's a pity, that the teenagers nowadays see, that these attributes aren't respected. The world of media and internet, from which the students get most of their information, prefers quick, easy-to-get, “instant” things, and don't relay the hard work behind the commanding achievements. I think in education we have to make stress not only to adapt ourselves to changing technical circumstances and matters, but also to put the negative changes of society to right path.

After setting the development papers into the course of maths lessons and testing the usage I'd like to assess the students once more. After comparing the results with the first one comes the re-working of exercise papers, or the following of usage. My aim is to work out such methods for developing competences, that can be fitted into curriculum, don't take extra time and efforts to use, don't "set back" the execution of given matters, can be used in general secondary school classes within the present circumstances, the working forms accuring variedly, and results increase in mathematical achievement too.

For motto I chose the words of a great educator, old, but still valid: "From wherever we see, the aim of our didactics must be to ferret out and hunt up the practise of education, so as to teachers should teach less, in the same time the students should learn more. In terms of this didactics let there be less confusion in schools, but more freedom, pleasure, and impresses a real development on all." (Comenius, 1657)

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